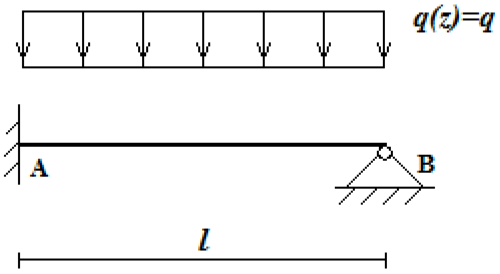
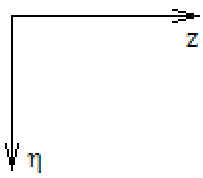
1. Risolvere il sistema iperstatico utilizzando <u>l'equazione della linea elastica</u>





Ricordando l'equazione differenziale della linea elastica del 4° ordine: $\eta^{IV}(z) = \dfrac{q(z)}{EI}$, si ha :

$\overline{AB}$

$0 \le z \le l$

$$EI \cdot \eta^{IV}(z) = q$$

$$EI \cdot \eta^{III}(z) = qz + c_1 = -T(z)$$

$$EI \cdot \eta^{II}(z) = q\frac{z^2}{2} + c_1 z + c_2 = -M(z)$$

$$EI \cdot \eta^{I}(z) = q\frac{z^3}{6} + c_1\frac{z^2}{2} + c_2 z + c_3 = -\varphi(z)$$

$$EI \cdot \eta(z) = q\frac{z^4}{24} + c_1\frac{z^3}{6} + c_2\frac{z^2}{2} + c_3 z + c_4$$

Impostando quindi le condizioni al contorno

$$\eta(A) = 0 \quad \rightarrow \quad \eta(0) = 0 \quad \rightarrow \quad c_4 = 0$$

$$\varphi(A) = 0 \quad \rightarrow \quad \varphi(0) = 0 \quad \rightarrow \quad c_3 = 0$$

$$\eta(B) = 0 \quad \rightarrow \quad \eta(l) = 0 \quad \rightarrow \quad q\frac{l^4}{24} + c_1\frac{l^3}{6} + c_2\frac{l^2}{2} + c_3 l + c_4 = 0$$

$$M(B) = 0 \quad \rightarrow \quad M(l) = 0 \quad \rightarrow \quad -q\frac{l^2}{2} - c_1 l - c_2 = 0$$

e risolvendo il sistema:

$$\begin{cases} c_4 = 0 \\ c_3 = 0 \\ q\dfrac{l^4}{24} + c_1\dfrac{l^3}{6} + c_2\dfrac{l^2}{2} + c_3 l + c_4 = 0 \\ c_2 = -q\dfrac{l^2}{2} - c_1 l \end{cases} \rightarrow \begin{cases} c_4 = 0 \\ c_3 = 0 \\ q\dfrac{l^4}{24} + c_1\dfrac{l^3}{6} + \left(-q\dfrac{l^2}{2} - c_1 l\right)\dfrac{l^2}{2} = 0 \\ c_2 = -q\dfrac{l^2}{2} - c_1 l \end{cases} \rightarrow \begin{cases} c_4 = 0 \\ c_3 = 0 \\ c_1 = -\dfrac{5}{8}ql \\ c_2 = q\dfrac{l^2}{8} \end{cases}$$

da cui riassumendo:

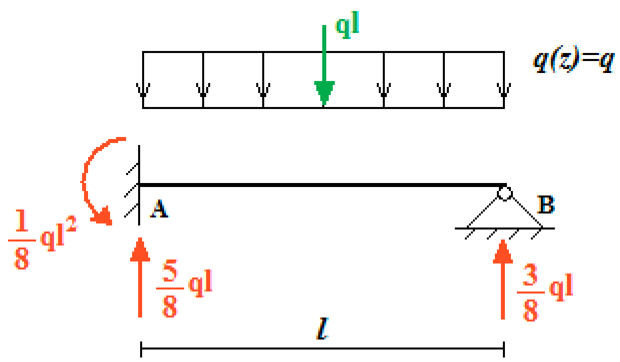
$\overline{AB}$

$0 \le z \le l$

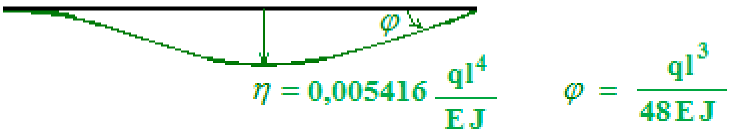
$$\begin{aligned} EI\cdot\eta^{IV}(z) &= q \\ EI\cdot\eta^{III}(z) &= qz - \frac{5}{8}ql = -T(z) \\ EI\cdot\eta^{II}(z) &= \frac{1}{2}qz^2 - \frac{5}{8}qlz + \frac{1}{8}ql^2 = -M(z) \\ EI\cdot\eta^{I}(z) &= \frac{1}{6}qz^3 - \frac{5}{16}qlz^2 + \frac{1}{8}ql^2 z = -\varphi(z) \\ EI\cdot\eta(z) &= \frac{1}{24}qz^4 - \frac{5}{48}qlz^3 + \frac{1}{16}ql^2 z^2 \end{aligned}$$

Le reazioni vincolari:

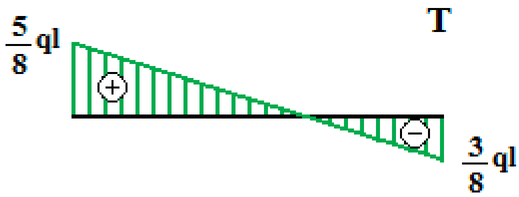
$$y(A) = T(A) = \frac{5}{8}ql \quad , \quad M(A) = -\frac{1}{8}ql^2 \quad , \quad y(B) = T(B) = -\frac{3}{8}ql$$

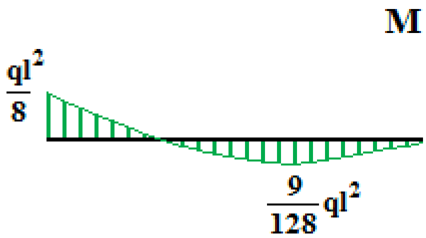


La valutazione della freccia:

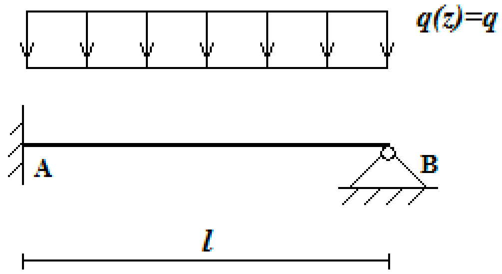


I relativi diagrammi

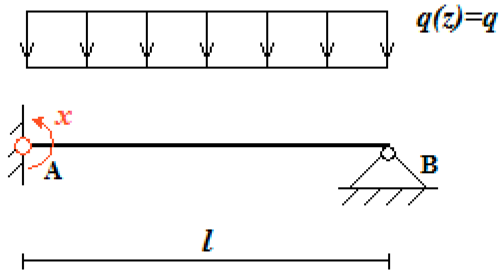




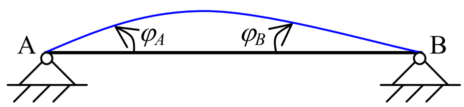
2. Risolvere il sistema iperstatico con il <u>metodo delle forze</u>



Introducendo una sconnessione semplice in A, si ottiene il sistema equivalente:

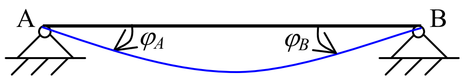


Ricordando quindi che, l'effetto momento in A porta a:



$$\varphi_A = +\frac{Ml}{3EI} \quad , \quad \varphi_B = -\frac{Ml}{6EI}$$

e che, l'effetto carico distribuito porta a:



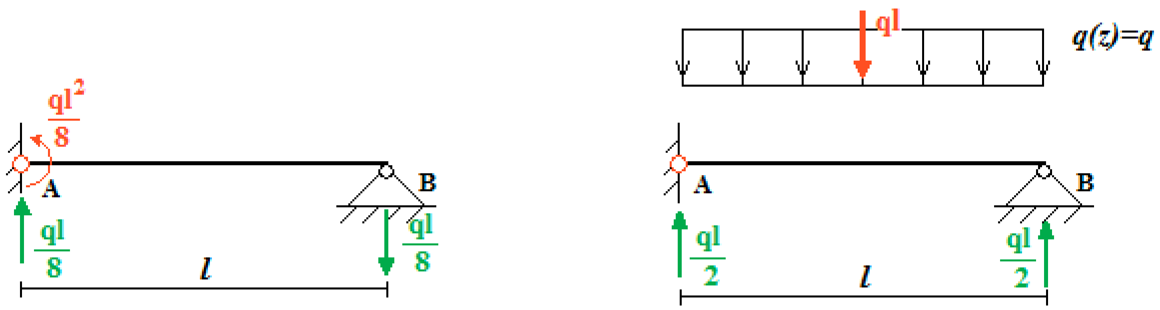
$$\varphi_A = -\frac{ql^3}{24EI} \quad , \quad \varphi_B = +\frac{ql^3}{24EI}$$

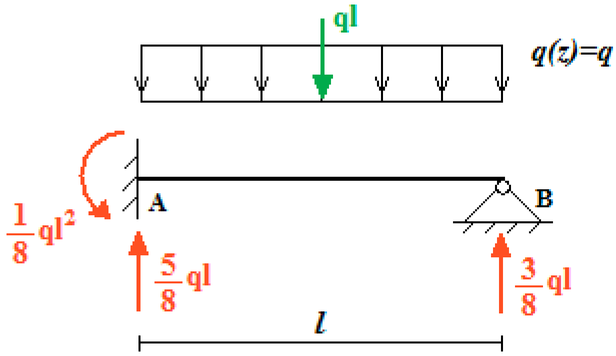
dall' equazione di congruenza, $\varphi_A = 0$

tramite il principio di sovrapposizione degli effetti (abbiamo convenzionalmente considerato le rotazioni positive in verso antiorario) si ha:

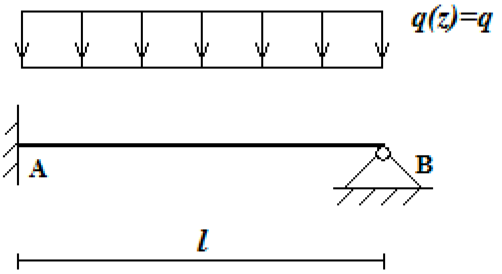
$$+\frac{xl}{3EI} - \frac{ql^3}{24EI} \quad = \quad 0 \quad \rightarrow \quad x = \frac{ql^2}{8}$$

dal calcolo dell'incognita iperstatica, si perviene poi alla risoluzione completa delle restanti reazioni vincolari (utilizzando nuovamente il P.S.E):

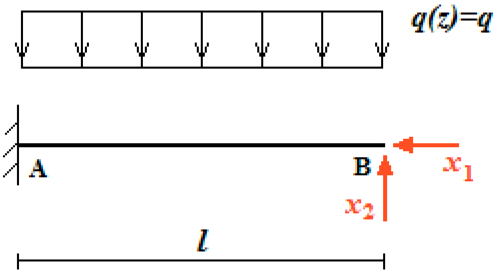




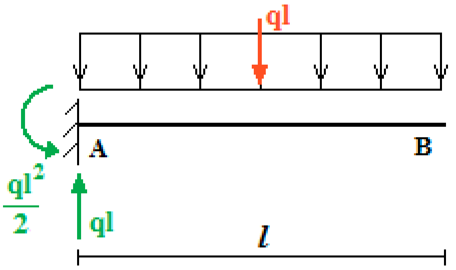
3. Risolvere il sistema iperstatico applicando il Principio dei Lavori Virtuali



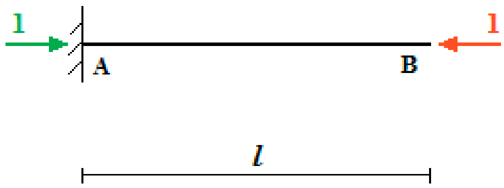
Dal relativo sistema equivalente:



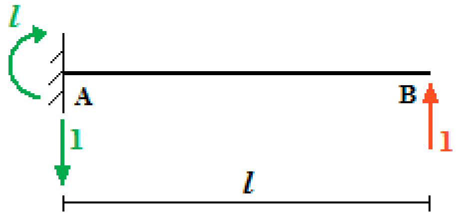
Sistema (0)



Sistema (1)



Sistema (2)



Impostando quindi a sistema , le equazioni di Mueller-Breslau:

$$\begin{cases} \eta_1 = \eta_{10} + x_1\eta_{11} + x_2\eta_{12} = 0 \\ \eta_2 = \eta_{20} + x_1\eta_{21} + x_2\eta_{22} = 0 \end{cases}$$

con :

$$\eta_{10} = \int_S \frac{M_0 M_1}{EI} ds = 0 \quad , \quad \eta_{11} = \int_S \frac{M_1 M_1}{EI} ds = 0 \quad , \quad \eta_{12} = \int_S \frac{M_1 M_2}{EI} ds = 0$$

$$\eta_{20} = \int_S \frac{M_0 M_2}{EI} ds = \int_0^l \frac{\left(qlz - \frac{ql^2}{2} - \frac{qz^2}{2}\right)(l-z)}{EI} dz = \int_0^l \frac{\left(\frac{qz^3}{2} - \frac{3qz^2}{2} + \frac{3qz}{2} - \frac{ql^3}{2}\right)}{EI} dz = -\frac{ql^4}{8EI}$$

$$\eta_{22} = \int_S \frac{M_2 M_2}{EI} ds = \int_0^l \frac{(l-z)^2}{EI} dz = \int_0^l \frac{\left(l^2 - 2lz + z^2\right)}{EI} dz = \frac{l^3}{3EI}$$

da cui si ha:

$$\begin{cases} 0 = x_1 \cdot 0 \\ 0 = -\frac{ql^4}{8EI} + x_2 \frac{l^3}{3EI} \end{cases} \rightarrow \begin{cases} x_1 = 0 \\ x_2 = \frac{3ql}{8EI} \end{cases}$$

N.B. Algebricamente la prima equazione è verificata per qualsiasi valore di x_1, in realtà il valore reale di x_1 è nullo dal momento che sul sistema iperstatico assegnato non gravano azioni risultanti con direzione orizzontale.

Le reazioni vincolari (utilizzando il P.S.E):

$$y(A) = y(A)_0 + x_1 y(A)_1 + x_2 y(A)_2 = +ql + \frac{3}{8} ql(-1) = \frac{5}{8} ql$$

$$M(A) = M(A)_0 + x_1 M(A)_1 + x_2 M(A)_2 = -\frac{ql^2}{2} + \frac{3}{8} ql(l) = -\frac{1}{8} ql^2$$

$$y(B) = y(B)_0 + x_1 y(B)_1 + x_2 y(B)_2 = +\frac{3}{8} ql(-1) = -\frac{3}{8} ql$$