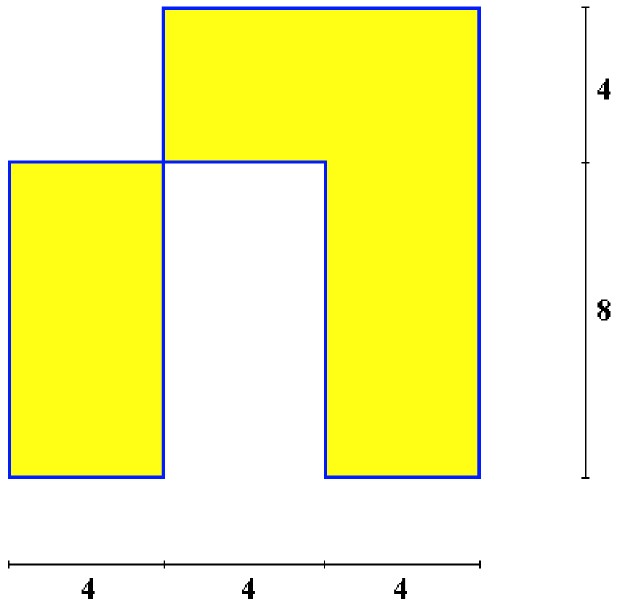
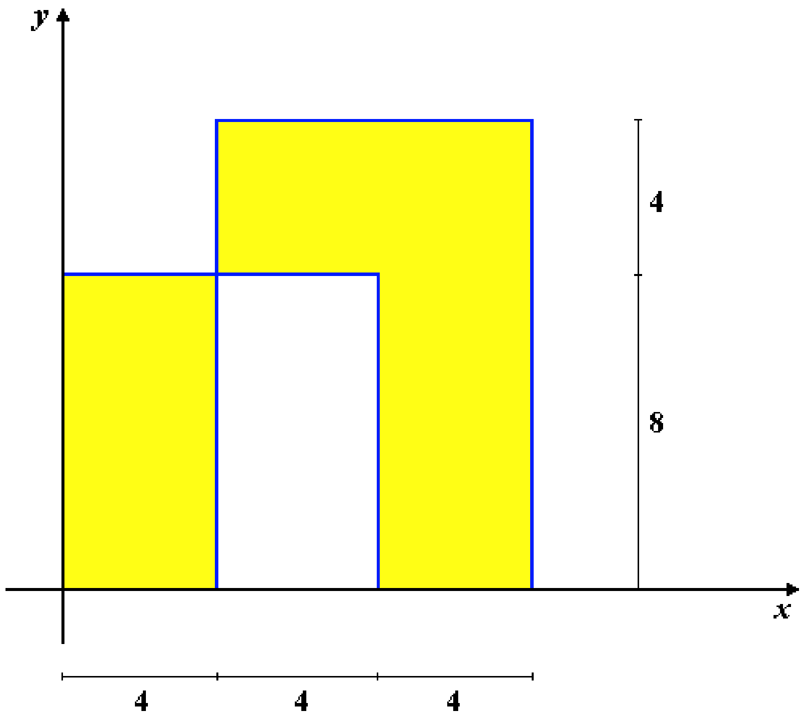
Determinare il baricentro del seguente sistema di masse (per il quale le misure sono espresse in cm).

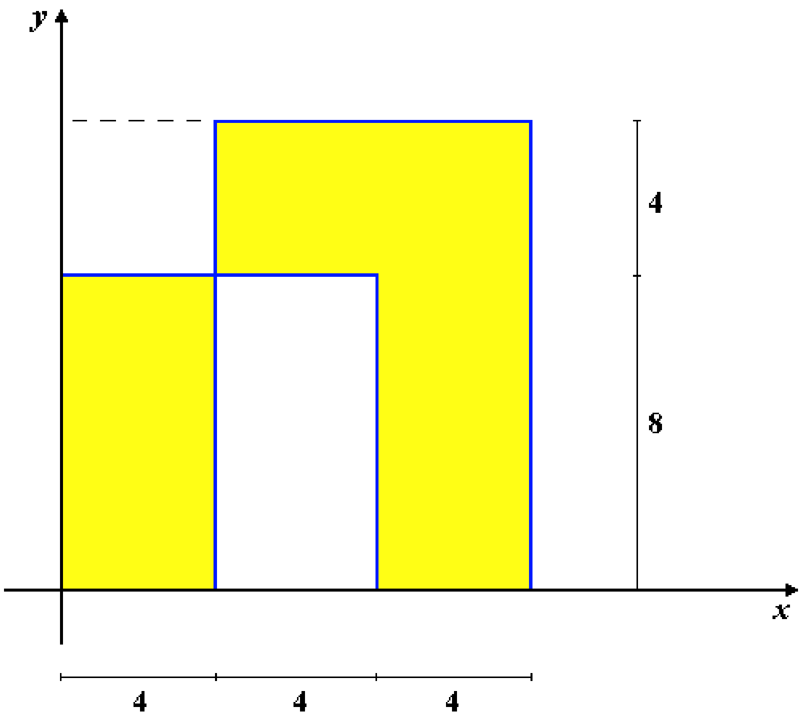


Tracciando un sistema di riferimento arbitrario 0xy :



Ricordando le formule del baricentro G , per sistemi continui di masse :

$$x_G = \frac{\int_A x dA}{\int_A dA} \quad , \quad y_G = \frac{\int_A y dA}{\int_A dA}$$



si ha :

$$x_G = \frac{\int_0^{12} 12x dx - \int_0^4 4x dx - \int_4^8 8x dx}{\int_0^{12} 12 dx - \int_0^4 4 dx - \int_4^8 8 dx} = \frac{\left(6x^2\right)_0^{12} - \left(2x^2\right)_0^4 - \left(4x^2\right)_4^8}{\left(12x\right)_0^{12} - \left(4x\right)_0^4 - \left(8x\right)_4^8} = \frac{864 - 32 - (256 - 64)}{144 - 16 - (64 - 32)} = \frac{640}{96} = 6{,}66\ cm$$

$$y_G = \frac{\int_0^{12} 12y dy - \int_8^{12} 4y dy - \int_0^8 4y dy}{\int_0^{12} 12 dy - \int_0^4 4 dy - \int_4^8 8 dy} = \frac{\left(6y^2\right)_0^{12} - \left(2y^2\right)_8^{12} - \left(2y^2\right)_0^8}{\left(12y\right)_0^{12} - \left(4y\right)_0^4 - \left(8y\right)_4^8} = \frac{864 - (288 - 128) - 128}{144 - 16 - (64 - 32)} = \frac{576}{96} = 6\ cm$$

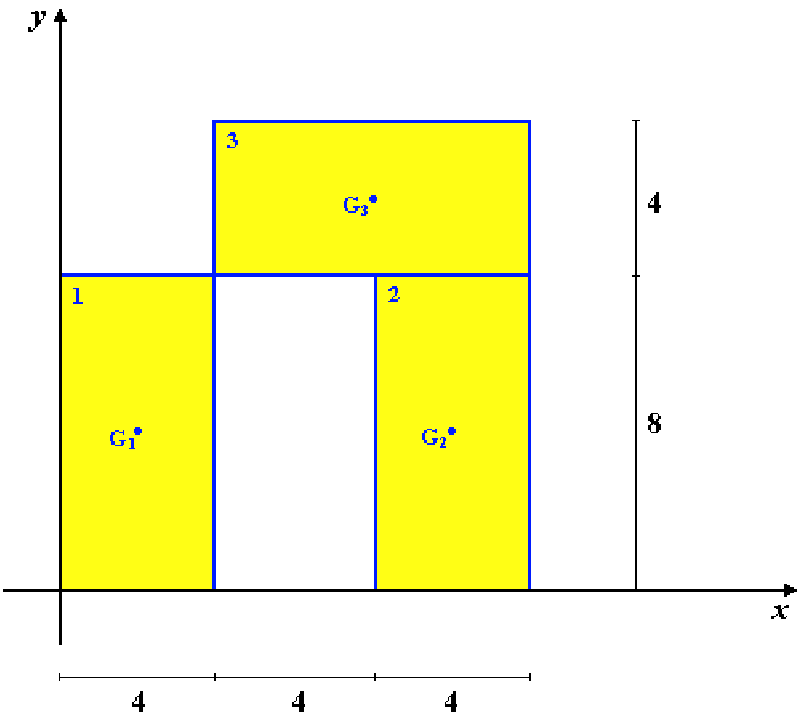
e quindi : $G\,(\,6{,}66\,;\,6\,)$

Più semplicemente si poteva procedere così :

Dalle formule pratiche del baricentro :

$$x_G = \frac{S_y}{A} \quad , \quad y_G = \frac{S_x}{A}$$

Scomponendo il sistema di masse , si ha :



$$x_G = \frac{S_{y(1)} + S_{y(2)} + S_{y(3)}}{A_1 + A_2 + A_3} \quad , \quad y_G = \frac{S_{x(1)} + S_{x(2)} + S_{x(3)}}{A_1 + A_2 + A_3}$$

con : $S_x = A \cdot y_G \quad , \quad S_y = A \cdot x_G$

$$x_G = \frac{32 \cdot 2 + 32 \cdot 10 + 32 \cdot 8}{32 + 32 + 32} = \frac{640}{96} = 6{,}66\ cm \quad , \quad y_G = \frac{32 \cdot 4 + 32 \cdot 4 + 32 \cdot 10}{32 + 32 + 32} = \frac{576}{96} = 6\ cm$$

Graficamente :