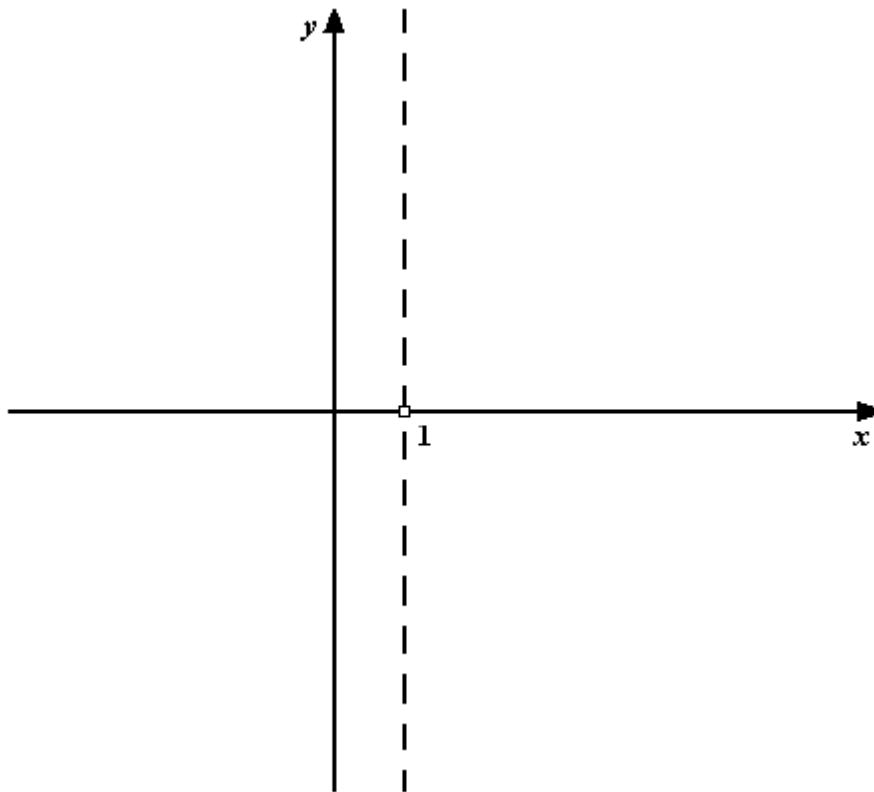


Studiare e rappresentare graficamente la funzione

$$f(x) = \frac{x^2 - 3x}{|x-1|}$$

1. Dominio

$$C.E. \Rightarrow \forall x \in \mathbb{R} : x \neq 1$$



2. Intersezioni Assi

$$\begin{cases} y = \frac{x^2 - 3x}{|x-1|} \\ x = 0 \end{cases} \Rightarrow \begin{cases} y = 0 \\ x = 0 \end{cases} \Rightarrow A(0, 0) \quad \text{punto d'intersezione con l'asse } y.$$

$$\begin{cases} \frac{x^2 - 3x}{|x-1|} = 0 \\ y = 0 \end{cases} \Rightarrow \begin{cases} x^2 - 3x = 0 \\ y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}, \begin{cases} x = 3 \\ y = 0 \end{cases} \Rightarrow B(3, 0)$$

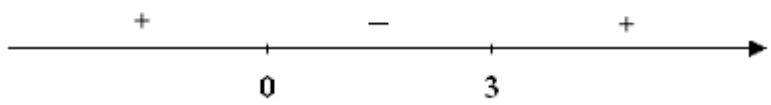
punto d'intersezione con l'asse x.

3. Segno della Funzione

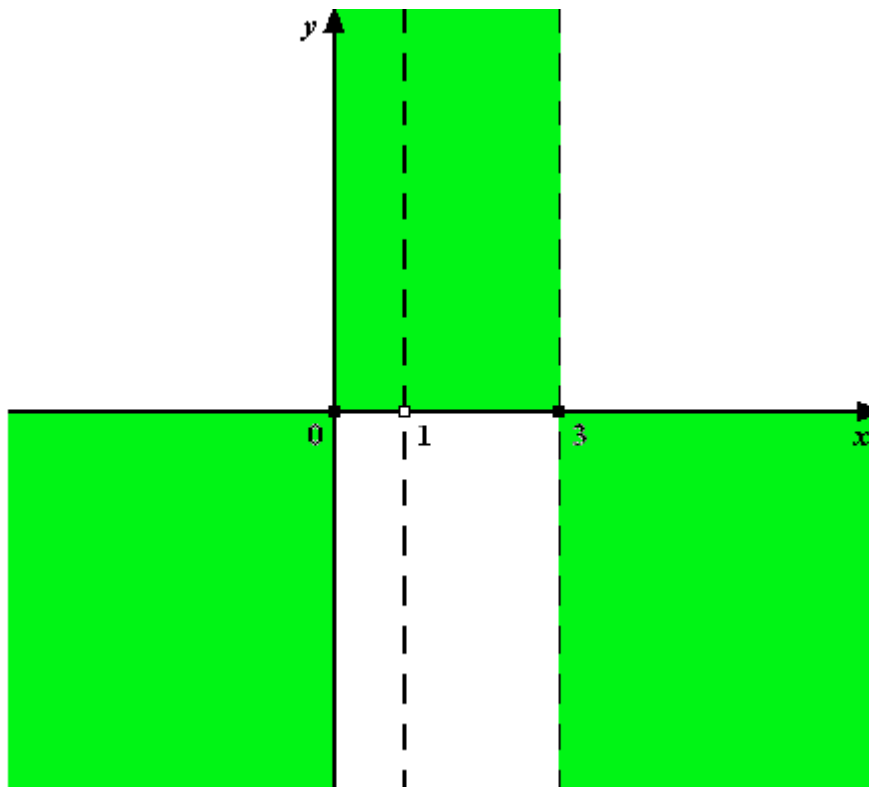
Vedi lez. 5 Algebra di Base

poichè la quantità al denominatore esprime sempre un valore positivo :

si ha : $x < 0$; $x > 3$ □



N.B. Le regioni piane contrassegnate dal colore verde escludono la presenza della funzione , avendone determinato sopra il segno.



4. Limiti

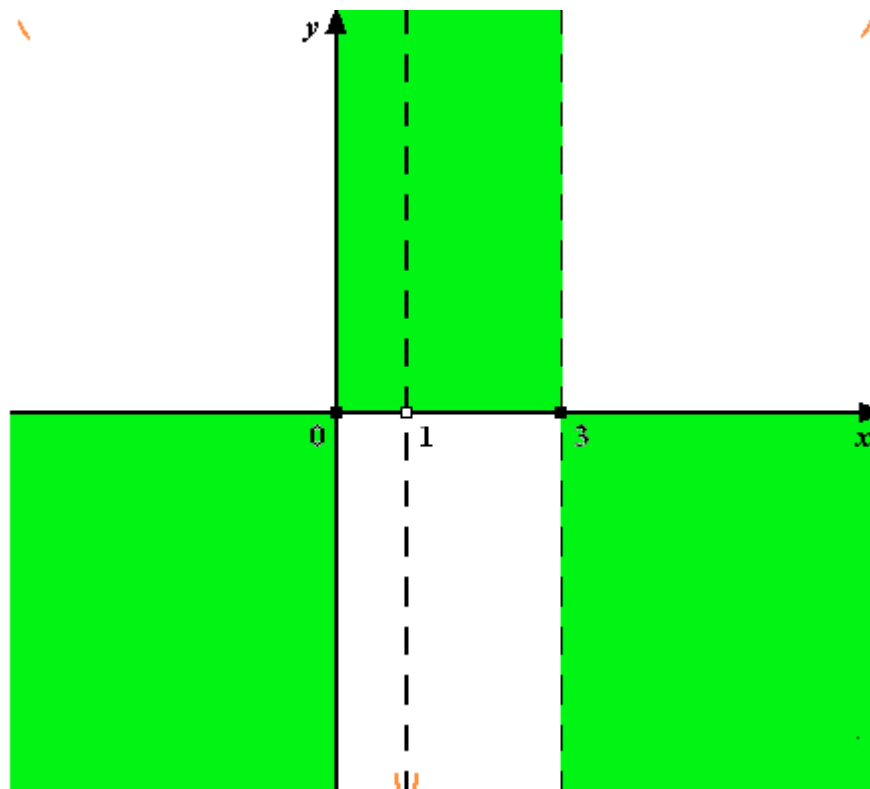
Vedi lez. 2 Studio di Funzione

$$\lim_{x \rightarrow 1} \frac{x^2 - 3x}{|x-1|} = -\infty$$

$$\lim_{x \rightarrow -\infty} \frac{x^2 - 3x}{|x-1|} = \left(\frac{+\infty}{+\infty} \right) \Rightarrow \lim_{x \rightarrow -\infty} \frac{x^2 - 3x}{1-x} = \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 - \frac{3}{x} \right)}{x^2 \left(\frac{1}{x^2} - \frac{1}{x} \right)} = +\infty$$

$$\lim_{x \rightarrow +\infty} \frac{x^2 - 3x}{|x-1|} = \left(\frac{+\infty - \infty}{+\infty} \right) \Rightarrow \lim_{x \rightarrow +\infty} \frac{x^2 - 3x}{x-1} = \lim_{x \rightarrow +\infty} \frac{x^2 \left(1 - \frac{3}{x} \right)}{x^2 \left(\frac{1}{x} - \frac{1}{x^2} \right)} = +\infty$$

La rappresentazione grafica dello studio dei limiti :



5. Asintoti

$$\boxed{x = 1} \quad \text{asintoto verticale}$$

verifica esistenza asymptoti obliqui : $y = mx + q$

$$m = \lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \left(\frac{+\infty}{-\infty} \right) \Rightarrow \lim_{x \rightarrow -\infty} \frac{x^2 - 3x}{x - x^2} = \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 - \frac{3}{x} \right)}{x^2 \left(\frac{1}{x} - 1 \right)} = -1$$

$$q = \lim_{x \rightarrow -\infty} f(x) - mx = \lim_{x \rightarrow -\infty} \frac{x^2 - 3x}{1 - x} + x = (+\infty - \infty) \Rightarrow \lim_{x \rightarrow -\infty} \frac{-2x}{1 - x} = \left(\frac{+\infty}{+\infty} \right)$$

$$q = \lim_{x \rightarrow -\infty} \frac{-2x}{x \left(\frac{1}{x} - 1 \right)} = 2$$

quindi per $x \rightarrow -\infty \Rightarrow y = -x + 2$ **asintoto obliquo** ,

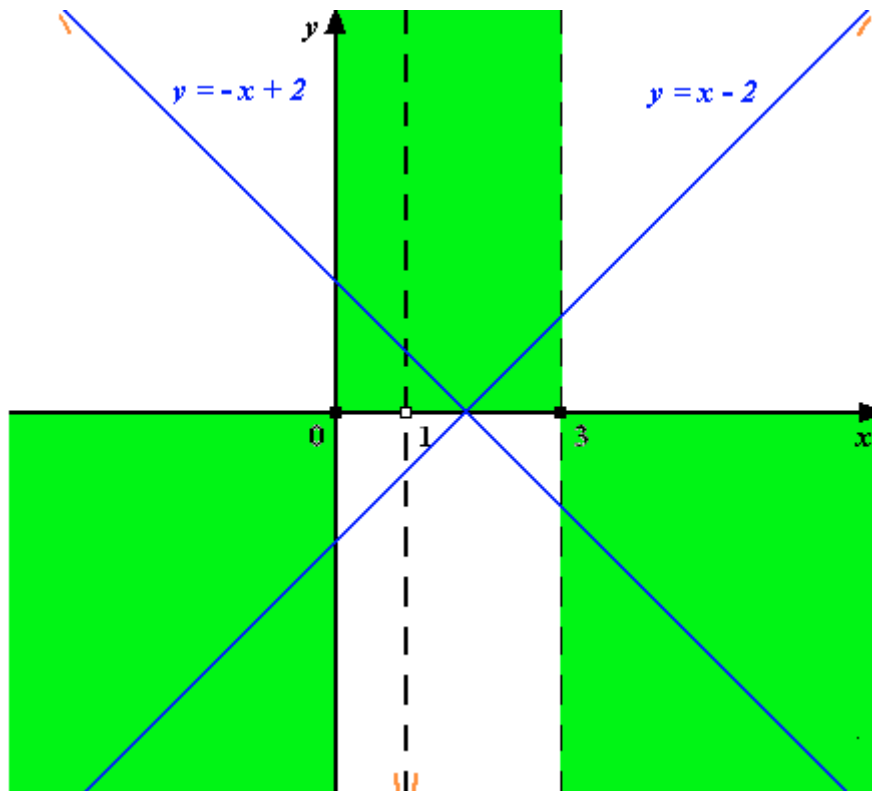
$$m = \lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \left(\frac{+\infty}{+\infty} \right) \Rightarrow \lim_{x \rightarrow +\infty} \frac{x^2 - 3x}{x^2 - x} = \lim_{x \rightarrow +\infty} \frac{x^2 \left(1 - \frac{3}{x} \right)}{x^2 \left(1 - \frac{1}{x} \right)} = 1$$

$$q = \lim_{x \rightarrow +\infty} f(x) - mx = \lim_{x \rightarrow +\infty} \frac{x^2 - 3x}{x - 1} - x = (+\infty - \infty) \Rightarrow \lim_{x \rightarrow +\infty} \frac{-2x}{x - 1} = \left(\frac{-\infty}{+\infty} \right)$$

$$q = \lim_{x \rightarrow +\infty} \frac{-2x}{x \left(1 - \frac{1}{x} \right)} = -2$$

quindi per $x \rightarrow +\infty \Rightarrow y = x - 2$ **asintoto obliquo**

La rappresentazione grafica dello studio degli asymptoti :



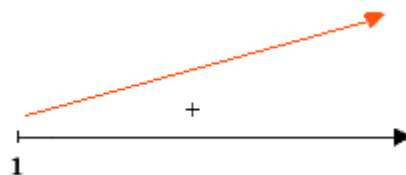
6. Derivata 1^a

Per $x - 1 > 0 \rightarrow x > 1 \Rightarrow f(x) = \frac{x^2 - 3x}{x - 1}$

$$f'(x) = \frac{(2x - 3)(x - 1) - (x^2 - 3x)}{(x - 1)^2} = \frac{x^2 - 2x + 3}{(x - 1)^2}$$

$$f'(x) > 0 \Rightarrow x^2 - 2x + 3 > 0 \Rightarrow \forall x \in \mathbb{R} : x > 1$$

Vedi lez. 3 Algebra di Base

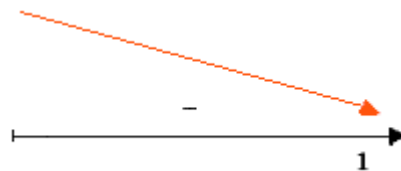


$$\text{Per } x-1 < 0 \quad \rightarrow \quad x < 1 \quad \Rightarrow \quad f(x) = \frac{x^2 - 3x}{1-x}$$

$$f'(x) = \frac{(2x-3)(1-x) + (x^2 - 3x)}{(1-x)^2} = \frac{-x^2 + 2x - 3}{(1-x)^2}$$

$$f'(x) > 0 \quad \Rightarrow \quad x^2 - 2x + 3 < 0 \quad \Rightarrow \quad \forall x \in \mathfrak{R}$$

Vedi lez. 3 Algebra di Base

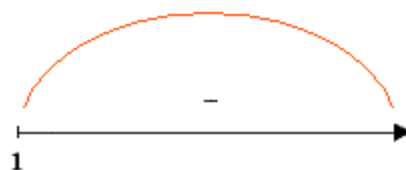


7. Derivata 2^a

$$\text{Per } x-1 > 0 \quad \rightarrow \quad x > 1 \quad \Rightarrow \quad f'(x) = \frac{x^2 - 2x + 3}{(x-1)^2}$$

$$f''(x) = \frac{(2x-2)(x-1)^2 - 2(x-1)(x^2 - 2x + 3)}{(x-1)^4} = \frac{2(x-1)(x^2 - 2x + 1 - x^2 + 2x - 3)}{(x-1)^4} = \frac{-4}{(x-1)^3}$$

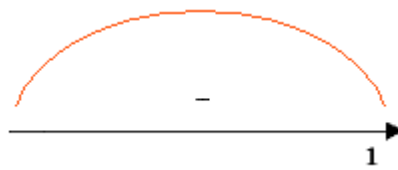
$$f''(x) > 0 \quad \Rightarrow \quad \forall x \in \mathfrak{R} : x > 1$$



Per $x-1 < 0 \rightarrow x < 1 \Rightarrow f'(x) = \frac{-x^2 + 2x - 3}{(1-x)^2}$

$$f''(x) = \frac{(-2x+2)(1-x)^2 + 2(1-x)(-x^2+2x-3)}{(1-x)^4} = \frac{-4}{(1-x)^3}$$

$$f''(x) > 0 \Rightarrow \forall x \in \mathfrak{R}$$



Il grafico :

