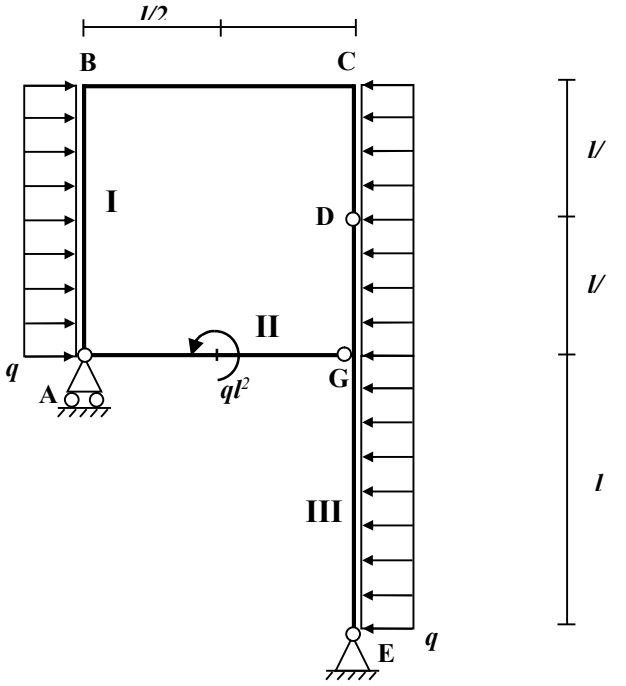
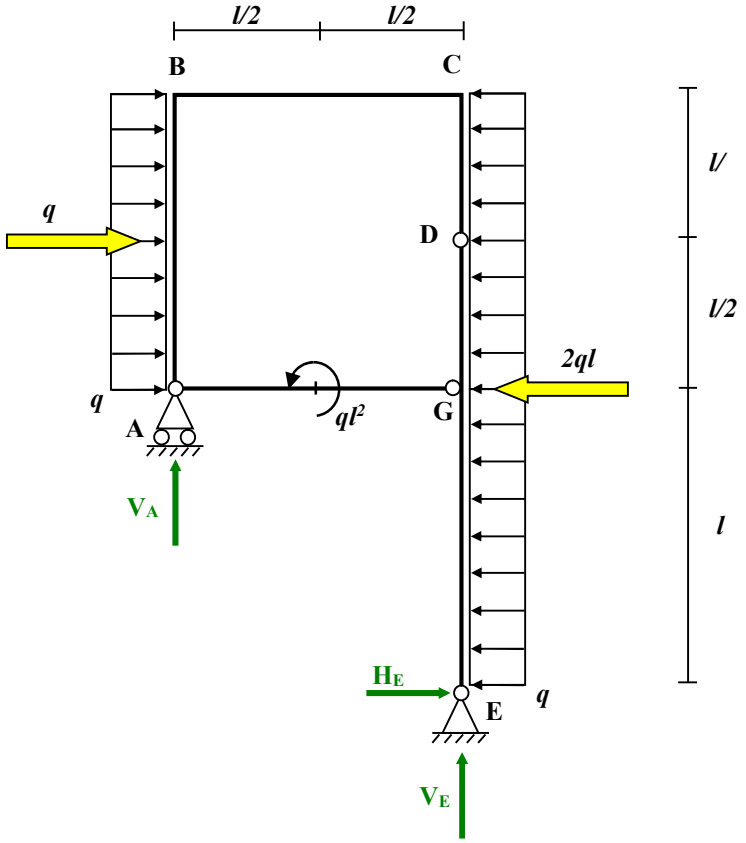
Determinare le reazioni vincolari e tracciare i diagrammi di sollecitazione

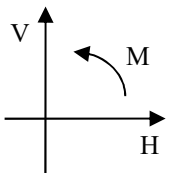


Calcolo delle reazioni vincolari :

Poiché la struttura esternamente è isostatica risolveremo le reazioni vincolari esterne :

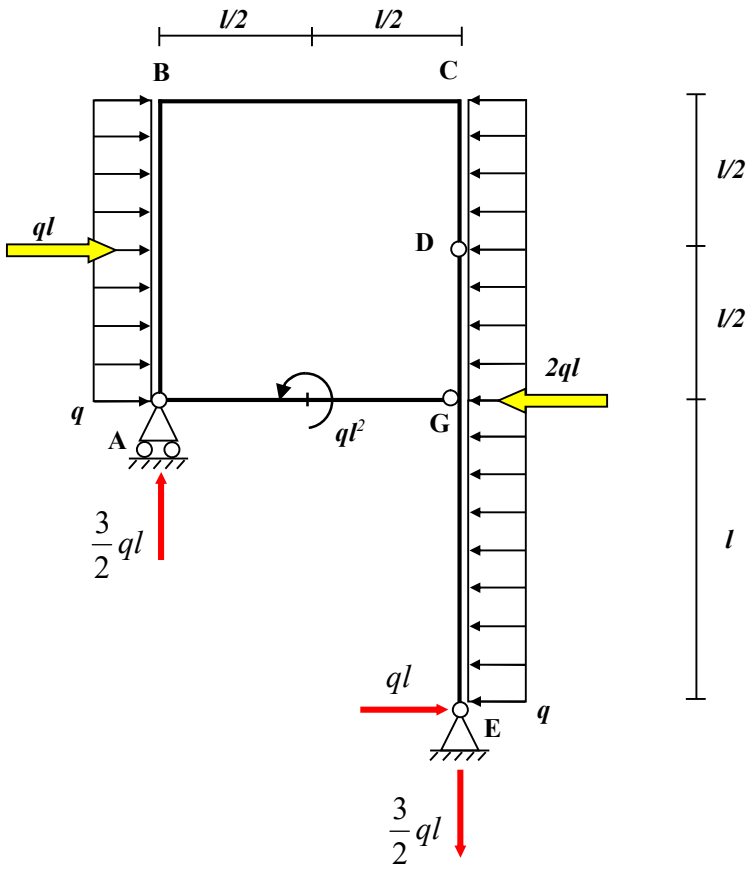


applicando le equazioni cardinali della statica si ha :

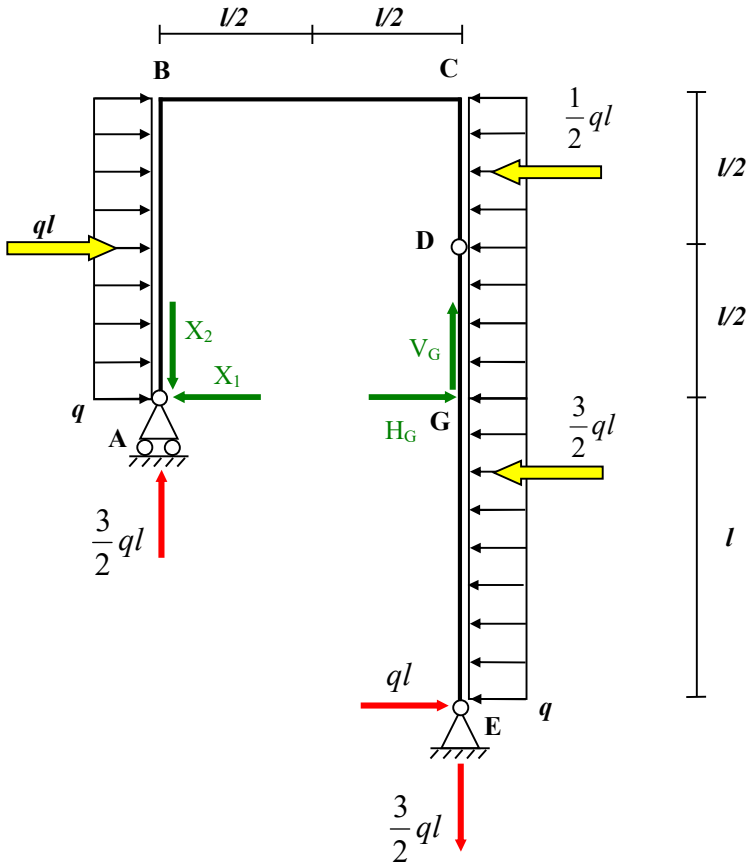


$$\begin{cases} \sum_H : \quad H_E + ql - 2ql = 0 \\ \sum_V : \quad V_A + V_E = 0 \\ \sum_M (E): \quad -V_A \cdot l + ql^2 + 2ql \cdot l - ql \cdot \frac{3}{2} l = 0 \end{cases} \Rightarrow \begin{cases} \sum_H : \quad H_E = +ql \\ \sum_V : \quad V_E = -\frac{3}{2} ql \\ \sum_M (E): \quad V_A = +\frac{3}{2} ql \end{cases}$$

Si ha quindi per il sistema equilibrato :



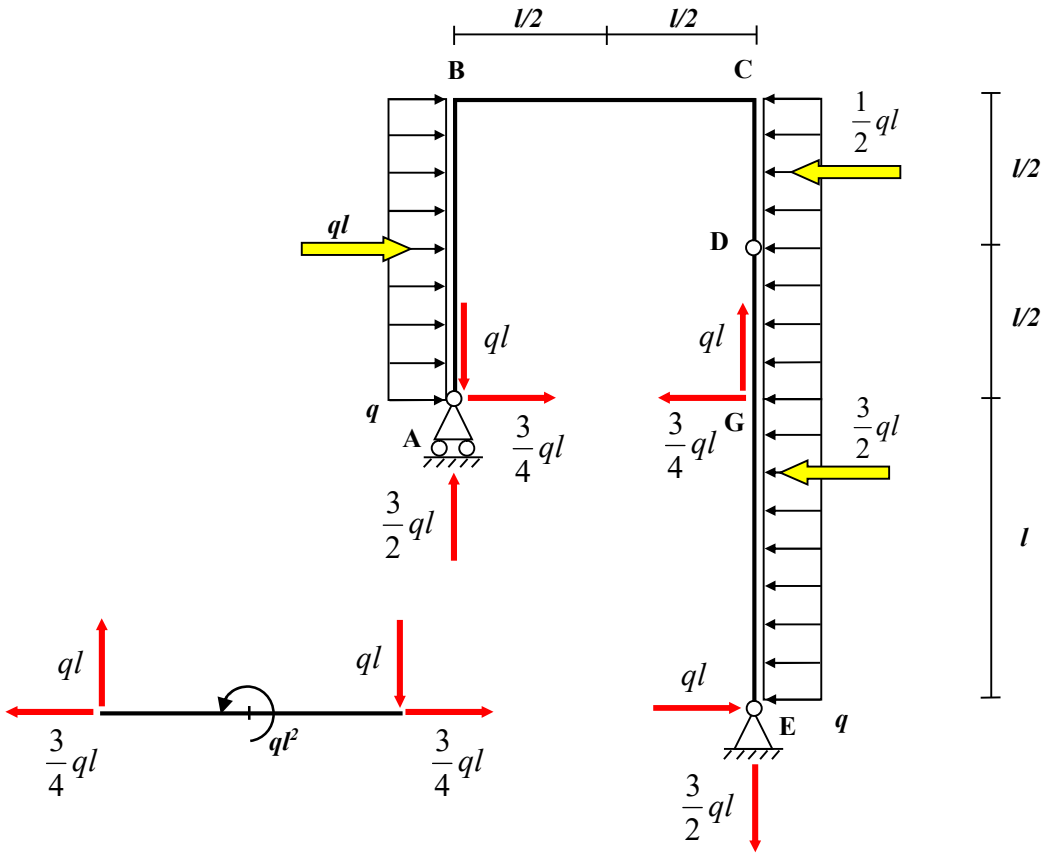
Calcoleremo ora la sottostruttura costituita dal I e dal III tronco, isolando il II tronco.



Applicandole tre equazioni cardinali della statica unitamente ad una equazione ausiliaria in D relativa al III tronco, si ha :

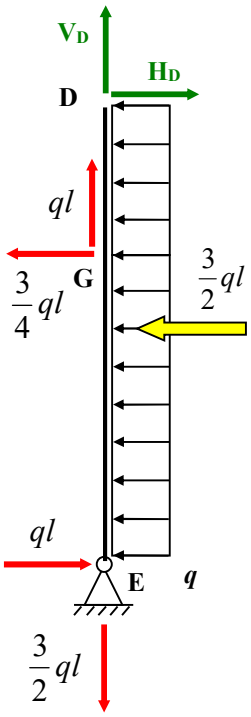
$$\begin{cases} \sum_H : \quad H_G - X_1 + ql - 2ql + ql = 0 \\ \sum_V : \quad + X_2 - V_G - \frac{3}{2}ql + \frac{3}{2}ql = 0 \\ \sum_M (A): \quad -ql \cdot \frac{1}{2}l + V_G \cdot l + ql \cdot l - \frac{3}{2}ql \cdot l = 0 \\ \sum_M (D)_{III} : \quad H_G \cdot \frac{l}{2} - \frac{3}{2}ql \cdot \frac{3}{4}l + ql \cdot \frac{3}{2}l = 0 \end{cases} \Rightarrow \begin{cases} \sum_H : \quad X_1 = -\frac{3}{4}ql \\ \sum_V : \quad X_2 = +ql \\ \sum_M (A): \quad V_G = +ql \\ \sum_M (D)_{III} : \quad H_G = -\frac{3}{4}ql \end{cases}$$

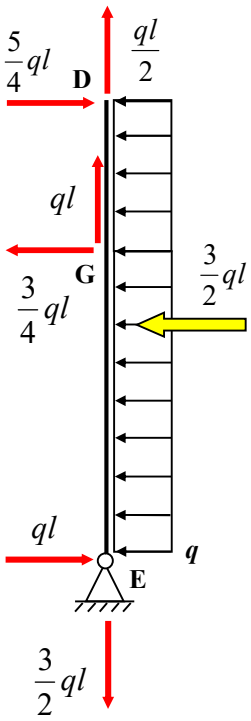
Si ha quindi per il sistema equilibrato :



Con 2 equazioni cardinali si calcolano sul III tronco le reazioni vincolari della cerniera in D:

$$\begin{cases} \sum_H : \quad H_D - \frac{3}{4}ql - \frac{3}{2}ql + ql = 0 \\ \sum_V : \quad V_D + ql - \frac{3}{2}ql = 0 \end{cases} \Rightarrow \begin{cases} \sum_H : \quad H_D = +\frac{5}{4}ql \\ \sum_V : \quad V_D = \frac{1}{2}ql \end{cases}$$





Diagrammi delle caratteristiche di sollecitazione . Applicando il metodo grafico :

N ql

Diagramma Sforzo Normale

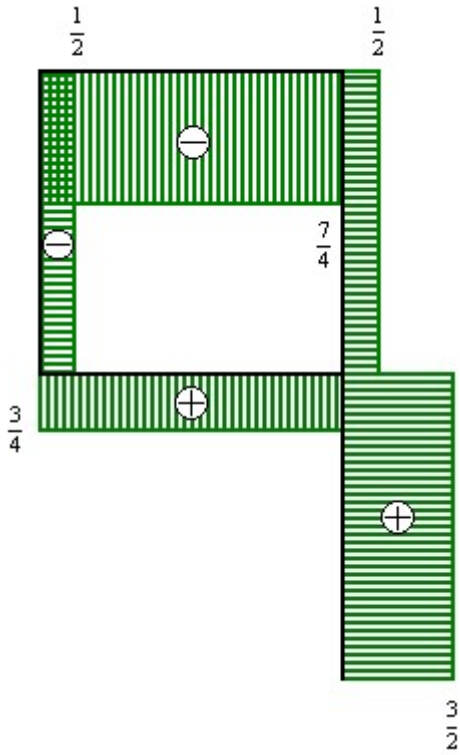


Diagramma Taglio **T** ql

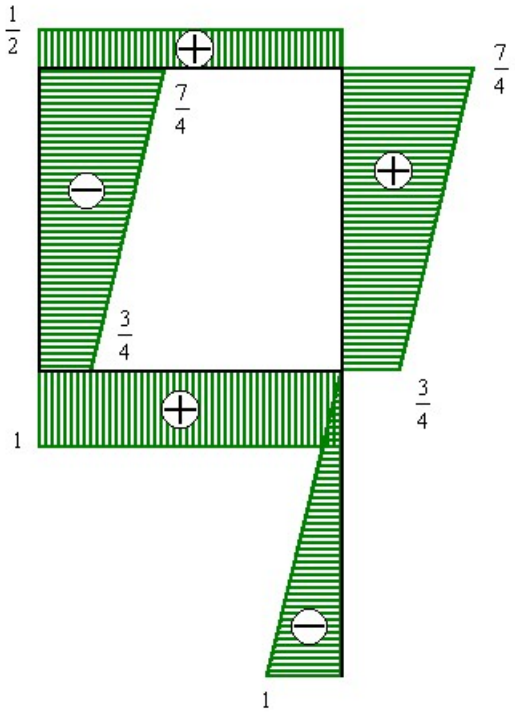
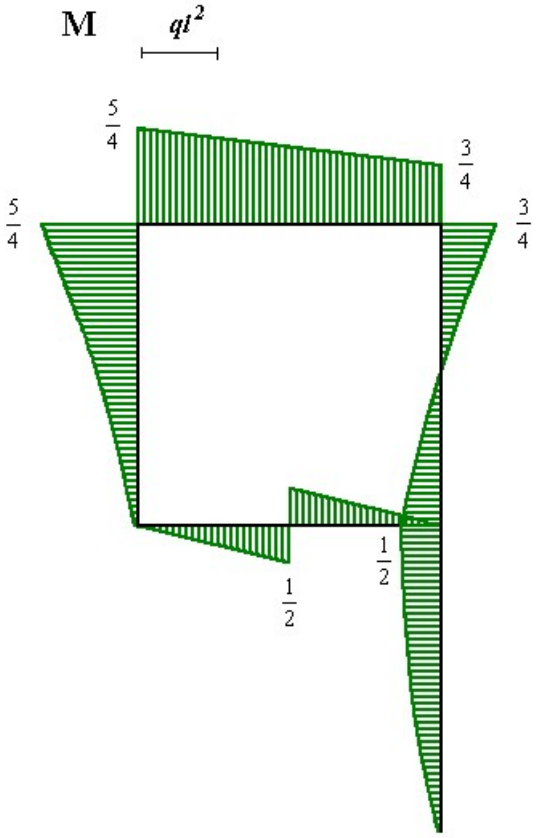
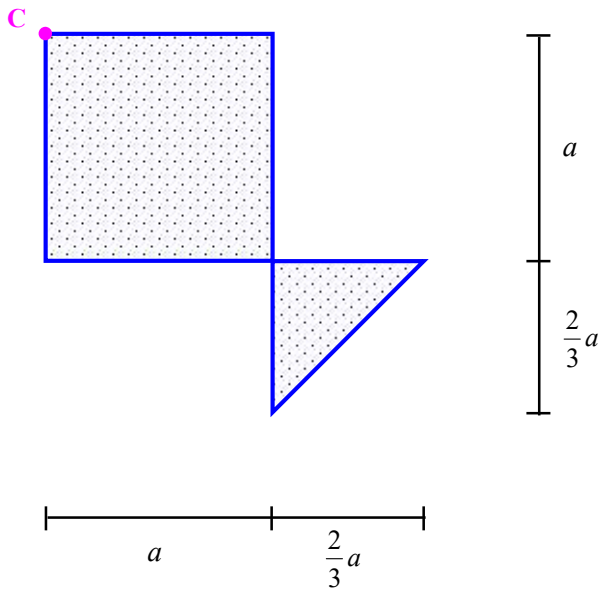


Diagramma Momento **M** ql^2

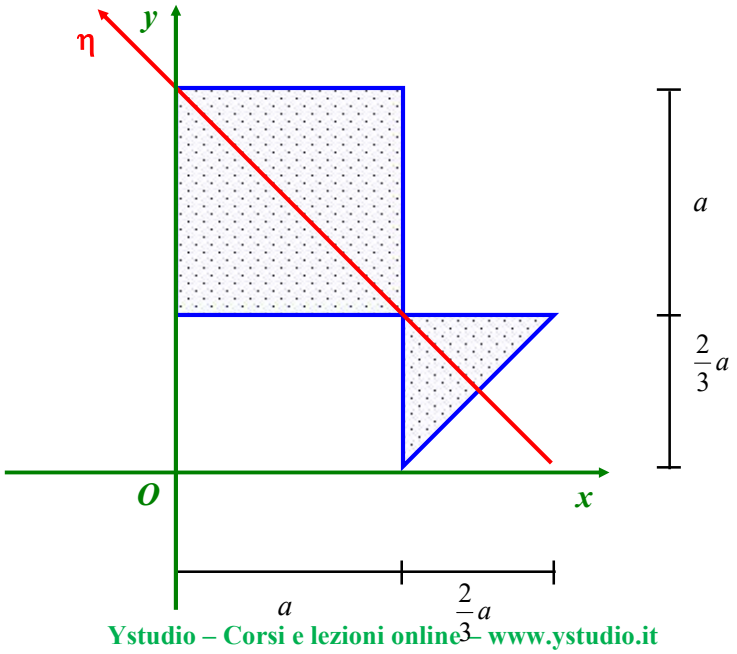


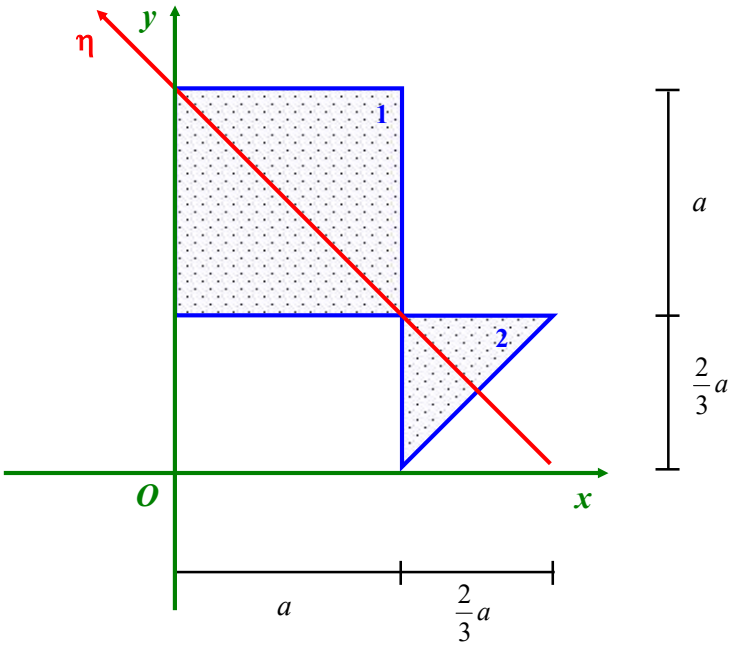
2. Dato il sistema di masse (aree) riportato in figura (zona puntinata), determinare la posizione della retta antipolare del punto C.



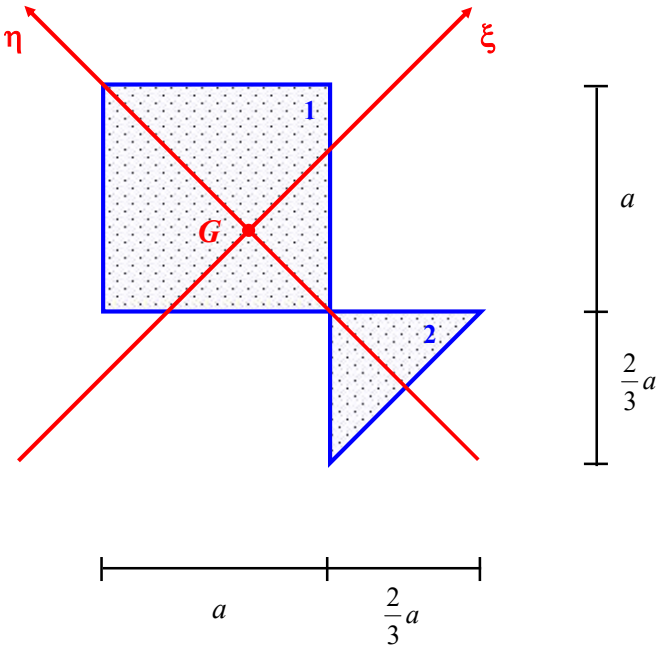
Svolgimento :

Poichè il sistema possiede un **asse di simmetria** inclinato a 45°questo oltre ad essere asse baricentrico del sistema è anche **asse principale d'inerzia** e quindi il baricentro ha coordinate uguali $x_G = y_G$ rispetto al sistema *Oxy*.





$$x_G = \frac{S_y}{A} \quad \Rightarrow \quad x_G = \frac{S_{y1} + S_{y2}}{A_1 + A_2} = \frac{a^2 \cdot \left(\frac{1}{2}a\right) + \frac{2}{9}a^2 \cdot \left(\frac{2}{9}a + a\right)}{a^2 + \frac{2}{9}a^2} = \frac{125}{198}a \ \ (0{,}63a)$$



Calcolando i momenti d'inerzia si ha :

$$I_\xi = I_{\xi 1} + I_{\xi 2} \quad \Rightarrow \quad \frac{a^4}{12} + a^2 \cdot \left(\frac{13\sqrt{2}}{99} a \right)^2 + \frac{\frac{2\sqrt{2}}{3} a \cdot \left(\frac{\sqrt{2}}{3} a \right)^3}{36} + \frac{2}{9} a^2 \cdot \left(\frac{11\sqrt{2}}{9} a - \frac{125\sqrt{2}}{198} a \right)^2 = 0{,}275 a^4$$

$$I_\eta = I_{\eta 1} + I_{\eta 2} \quad \Rightarrow \quad \frac{a^4}{12} + \frac{\frac{\sqrt{2}}{3} a \cdot \left(\frac{2\sqrt{2}}{3} a \right)^3}{48} = 0{,}091 a^4$$

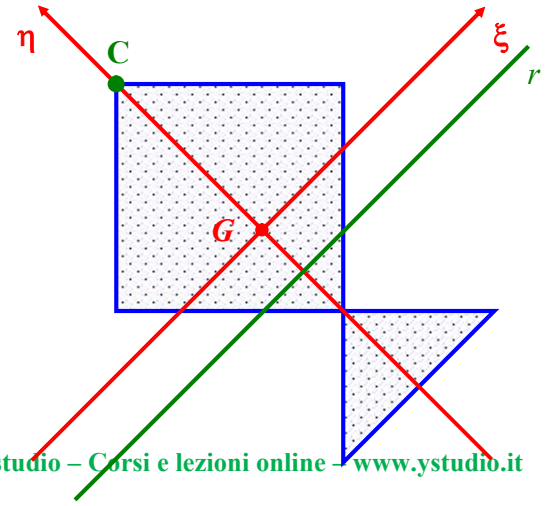
I relativi raggi giratori d'inerzia :

$$\rho^2{}_\xi = \frac{I_\xi}{A} = 0{,}275 a^4 \cdot \frac{0{,}818}{a^2} = 0{,}225 a^2$$

$$\rho^2{}_\eta = \frac{I_\eta}{A} = 0{,}091 a^4 \cdot \frac{0{,}818}{a^2} = 0{,}074 a^2$$

La retta antipolare assume come equazione : $\frac{a}{c}\xi + \frac{b}{c}\eta + 1 = 0$, e poiché le coordinate di C rispetto ad $O\xi\eta$ sono date da : $C(\,0\,;0{,}88a\,)$ ricordando le relazioni che legano il centro relativo con la retta $\frac{\xi_R}{\rho_\eta^{\,2}} = \frac{a}{c}$; $\frac{\eta_R}{\rho_\xi^{\,2}} = \frac{b}{c}$, si ha :

$\frac{a}{c} = 0$; $\frac{b}{c} = 0{,}88a \cdot \frac{1}{0{,}225a^2} = \frac{3{,}9}{a}$ $\Rightarrow$ $\frac{3{,}9}{a}\eta + 1 = 0$ equazione retta antipolare.



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